9. FELSEN L. and MARKUVITZ N., Wave radiation and scattering /Russian translation/, 1, Moscow, Mir, 1978.
10. MOKRIK R.I. and PYR'EV YU.A., Analytic properties of the characteristic parameters of generalized thermoelasticity. Mathematical methods and physical-mathematical fields. Sb. statei, Kiev, Nauk. Dumka, 15, 1982.
11. GEL'FAND I.M. and SHILOV G.E., Some problems of the theory of differential equations. Moscow, Fizmatgiz, 1958.
12. WHITHAM G.B., Some comments on wave propgation and shock wave structure with application to magnetohydrodynamics. Communs Pure Appl. Math., 12, 1, 1959.
13. MOKRIK R.I. and PYR'EV YU.A., Properties of the solutions of the dynamic problems of generalized coupled thermoelasticity, PMM, 45, 5, 1981.
14. POPOV E.B., The dynamic coupled thermoelasticity probelm for a half-space ignoring the finite velocity of heat propagation. PMM, 31, 2, 1967.
15. DILLON O.W., Thermoelasticity when the material coupling parameter equals unity. Trans. ASME Ser. E.J. Appl. Mech., 32, 2, 1965.
16. GRIBANOV V.F. and PANICHKIN N.G., Investigation of the effect of thermomechanical interaction by the method of finite elements. Prikl. Mekhan., 15, 6, 1979.
17. SHATALOV A.G., Influence of the relaxation time of heat flow on the wave stresses in a semi-infinite medium, Fizika i khimiya kondensirovannykh sred. Voronezh, Izd-vo Voronezh. politekhn. in-ta, 1981.
18. NIKEL R.E. and SACKMAN J.L., Approximate solutions in linear, coupled thermoelasticity. Trans. ASME. Ser. E.J. Appl. Mech., 35, 2, 1968.
19. TING E.C. and CHEN H.C., A unified numerical approach for thermal stress waves. Comput. and Struct. 15, 2, 1982.

Translated by W.C.

PMM U.S.S.R., Vol.49, No.6, pp. 719-724, 1985
Printed in Great Britain

0021-8928/85 $10.00+0.00
Pergamon Journals Ltd.

REDUCTION OF THREE-DIMENSIONAL DYNAMICAL ELASTICITY THEORY PROBLEMS WITH ARBITRARILY LOCATED PLANE SLITS TO INTEGRAL EQUATIONS*

V.V. MYKHAS'KIV and M.V. KHAI

By generalizing a method described earlier /1/ for reducing three-dimensional dynamical problems of elasticity theory for a body with a slit to integral equations, integral equations are obtained for an infinite body with arbitrarily located plane slits. The interaction of disc-shaped slits located in one plane is investigated when normal external forces that vary sinusoidally with time (steady vibrations) are given on their surfaces.

Problems of the reduction of dynamical three-dimensional elasticity theory problems to integral equations for an infinite body weakened by a plane slit were examined in /1, 2/. The solution of the initial problem is obtained in /1/ by applying a Laplace integral transform in time to the appropriate equations and constructing the solution in the form of Helmholtz potentials with densities characterizing the opening of the slit during deformation of the body. The problem under consideration is solved in /2/ by using the fundamental Stokes solution /3/ with subsequent construction of the solution in the form of an analogue of the elastic potential of a double layer.

1. We consider an ealstic infinite body weakened by plane arbitrarily located slits whose opposite surfaces S_n^+ and S_n^- $(n = 1, 2, \ldots, N)$ are subjected to selfequilibrated external forces varying with time t. We consider the initial conditions of the problem to be zero.

We select a basic Cartesian coordinate system $Ox_1x_2x_3$ with origin O at an arbitrary point of the body and local coordinate systems $O_nx_{1n}x_{2n}x_{3n}$ $(n = 1, 2, \ldots, N)$ in such a way that the domain S_n which the n-th slit occupies would be contained in the coordinate plane $x_{1n}O_nx_{2n}$, while the values $x_{3n} = \pm 0$ (Fig.1) would correspond to the surfaces $S_n^\pm$. Let x denote the point with coordinates (x_1, x_2, x_3).

*Prikl.Matem.Mekhan., 49, 6, 943-950, 1985

To reduce the initial problem to integral equations we use the fact that the opposite surfaces of the slit are displaced realtive to each other under the action of external loads. If $\beta_{jn}(t, x)$ denotes functions that characterize the displacement of opposite points of the n-th slit surface along the coordinate axes $O_n x_{jn}$, then the stress σ_{ljn} in the n-th local coordinate system is determined by the relationships

$$\sigma_{ljn}(t, x_n) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \sigma^*_{ljn}(p, x_n) \exp(pt)\, dp \quad (l, j = 1, 2, 3)$$

where the components σ^*_{ljn} of the stress in the Laplace transform can be written as follows /1/:

$$\frac{\sigma^*_{33n}}{2G} = \frac{2}{\omega_2^2}\left[\Delta_n(\Delta_n - \omega_2^2)(F_{3n} - P_{3n}) + \frac{\omega_2^4}{4} F_{3n} + \left(\Delta_n - \frac{\omega_2^2}{2}\right)\frac{\partial W_n}{\partial x_{3n}}\right] \tag{1.1}$$

$$\frac{\sigma^*_{j3n}}{2G} = \frac{2}{\omega_2^2}\Big\{(-1)^j \Big[\left(\Delta_n - \frac{\omega_2^2}{2}\right)^2 P_{(3-j)n} - \Delta_n(\Delta_n - \omega_1^2) F_{(3-j)n} -$$
$$\left(\Delta_n - \frac{3\omega_2^2}{4}\right) \frac{\partial}{\partial x_{(3-j)n}} \left(\frac{\partial P_{2n}}{\partial x_{2n}} + \frac{\partial P_{1n}}{\partial x_{1n}}\right) +$$
$$(\Delta_n - \omega_1^2) \frac{\partial}{\partial x_{(3-j)n}} \left(\frac{\partial F_{2n}}{\partial x_{2n}} + \frac{\partial F_{1n}}{\partial x_{1n}}\right) -$$
$$\left(\Delta_n - \frac{\omega_2^2}{2}\right) \frac{\partial^2 (F_{3n} - P_{3n})}{\partial x_{jn} \partial x_{3n}}\Big\}$$

$$\frac{\sigma^*_{12n}}{2G} = \frac{2}{\omega_2^2}\Big\{-(\Delta_n - \omega_2^2)\frac{\partial^3 (F_{3n} - P_{3n})}{\partial x_{1n} \partial x_{2n}} - \frac{\omega_2^2}{2}\Big[\frac{\partial^2 F_{3n}}{\partial x_{1n} \partial x_{2n}} -$$
$$\frac{\partial}{\partial x_{3n}}\left(\frac{\partial P_{1n}}{\partial x_{1n}} - \frac{\partial P_{2n}}{\partial x_{2n}}\right)\Big] - \frac{\partial^3 W_n}{\partial x_{1n} \partial x_{2n} \partial x_{3n}}\Big\}$$

$$\frac{\sigma^*_{jjn}}{2G} = \frac{2}{\omega_2^2}\Big\{-\left(\Delta_n - \frac{\omega_2^2}{2}\right)\left(\frac{\nu\omega_1^2}{1-2\nu} - \frac{\partial^2}{\partial x_{jn}^2}\right) F_{3n} +$$
$$(\Delta_n - \omega_2^2)\frac{\partial^2 P_{3n}}{\partial x_{jn}^2} + \frac{\partial}{\partial x_{3n}}\Big[(-1)^j \frac{\omega_2^2}{2} \frac{\partial P_{(3-j)n}}{\partial x_{jn}} -$$
$$\frac{\nu\omega_1^2}{1-2\nu}\left(\frac{\partial F_{2n}}{\partial x_{1n}} - \frac{\partial F_{1n}}{\partial x_{2n}}\right) - \frac{\partial^2 W_n}{\partial x_{jn}^2}\Big]\Big\}$$

$$W_n = \frac{\partial (F_{2n} - P_{2n})}{\partial x_{1n}} - \frac{\partial (F_{1n} - P_{1n})}{\partial x_{2n}}, \quad \Delta_n = \frac{\partial^2}{\partial x_{1n}^2} + \frac{\partial^2}{\partial x_{2n}^2}$$

$$F_{jn} = \iint_{S_n} \beta^*_{jn}(p, \xi)\, \Phi_1(x_n, \xi)\, d_\xi S, \quad P_{jn} = \iint_{S_n} \beta^*_{jn}(p, \xi)\, \Phi_2(x_n, \xi)\, d_\xi S$$

$$\Phi_j(x_n, \xi) = \frac{\exp(-\omega_j |x_n - \xi|)}{|x_n - \xi|}, \quad \omega_j = \frac{p}{c_j}$$

Here x_n is the point with coordinates (x_{1n}, x_{2n}, x_{3n}) in the n-th coordinate system, p is the Laplace transform parameter, where $\operatorname{Re} p > 0$, c_1, c_2 are the longitudinal and transverse wave propagation velocities, G is the shear modulus, and ν is Poisson's ratio. The asterisk denotes that the Laplace transform of the appropriate functions in t is considered.

Since there are N plane arbitrarily located slits in the body under consideration, the stresses at any point of the body equal the sum of the stresses due to the opening of all the slits during body deformation.

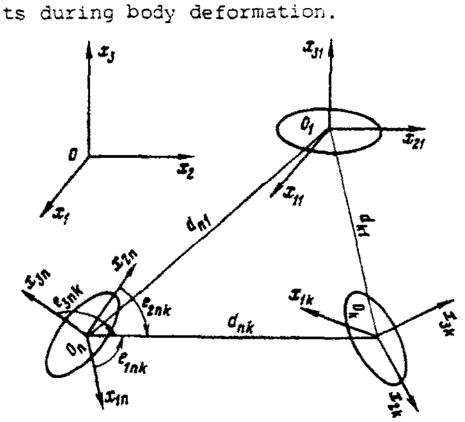

Fig.1

Let $l_{jkn}, m_{jkn}, n_{jkn}$ denote the direction cosines of the axes $O_n x_{jn}$ in the coordinate system $O_k x_{1k} x_{2k} x_{3k}$ which are given as follows:

	$O_n x_{1n}$	$O_n x_{2n}$	$O_n x_{3n}$
$O_k x_{1k}$	l_{1kn}	l_{2kn}	l_{3kn}
$O_k x_{2k}$	m_{1kn}	m_{2kn}	m_{3kn}
$O_k x_{3k}$	n_{1kn}	n_{2kn}	n_{3kn}

Then the stress due to displacement of the surfaces of the k-th slit are determined at the side of n-th slit location by the formulas

$$N^*_{jkn} = \sigma^*_{11k} l_{j3kn} + \sigma^*_{22k} m_{j3kn} + \sigma^*_{33k} n_{j3kn} + \sigma^*_{12k} l^{(1)}_{j3kn} +$$
$$\sigma^*_{23k} m^{(1)}_{j3kn} + \sigma^*_{13k} n^{(1)}_{j3kn} \quad (j = 1, 2, 3)$$
$$l_{j3kn} = l_{jkn} l_{3kn}, \quad m_{j3kn} = m_{jkn} m_{3kn}$$

$$n_{j3kn} = n_{jkn} n_{3kn}, \quad l_{j3kn}^{(1)} = l_{jkn} m_{3kn} + l_{3kn} m_{jkn}$$
$$m_{j3kn}^{(1)} = m_{jkn} n_{3kn} + m_{3kn} n_{jkn}, \quad n_{j3kn}^{(1)} = n_{jkn} l_{3kn} + n_{3kn} l_{jkn}$$

Summing all the stresses at the location of the n-th slit that are due to the displacement of opposite surfaces of the slits, and equating them to the given external $N_{jn}(t, x)$ forces (N_{3n} are normal and N_{1n}, N_{2n} tangential), we obtain a system of $3N$ integral equations to determine the functions $\beta_{jn}(t, x)$. This system takes the following form in Laplace transforms:

$$\Delta_n(\Delta_n - \omega_2^2) \iint_{S_n} \beta_{3n}^*(p, \xi)[\Phi_1(x_n, \xi) - \Phi_2(x_n, \xi)]\, d_\xi S + \tag{1.2}$$
$$\frac{\omega_2^4}{4} \iint_{S_n} \beta_{3n}^*(p, \xi)\Phi_1(x_n, \xi)\, d_\xi S + \sum_{k=1}^{N}{}' \iint_{S_k} \sum_{s=1}^{3} \beta_{sk}^*(p, \xi) \times$$
$$\chi_{3skn}(p, \xi, x_n)\, d_\xi S = -\frac{\omega_2^2}{4G} N_{3n}^*(p, x_n)$$
$$\Delta_n(\Delta_n - \omega_2^2) \iint_{S_n} \beta_{(3-j)n}^*(p, \xi)[\Phi_1(x_n, \xi) - \Phi_2(x_n, \xi)]\, d_\xi S +$$
$$(\omega_2^2 - \omega_1^2)\Delta_n \iint_{S_n} \beta_{(3-j)n}^*(p, \xi)\Phi_1(x_n, \xi)\, d_\xi S - \frac{\omega_2^4}{4} \iint_{S_n} \beta_{(3-j)n}^*(p, \xi)\Phi_2(x_n, \xi)\, d_\xi S -$$
$$(\Delta_n - \omega_1^2)\frac{\partial}{\partial x_{(3-j)n}} \iint_{S_n} \left[\beta_{1n}^*(p, \xi)\frac{\partial \Phi_1(x_n, \xi)}{\partial x_{1n}} + \right.$$
$$\left. \beta_{2n}^*(p, \xi)\frac{\partial \Phi_1(x_n, \xi)}{\partial x_{2n}}\right] d_\xi S + \left(\Delta_n - \frac{3\omega_2^2}{4}\right) \times$$
$$\frac{\partial}{\partial x_{(3-j)n}} \iint_{S_n} \left[\beta_{1n}^*(p, \xi)\frac{\partial \Phi_2(x_n, \xi)}{\partial x_{1n}} + \beta_{2n}^*(p, \xi)\frac{\partial \Phi_2(x_n, \xi)}{\partial x_{2n}}\right] d_\xi S -$$
$$\sum_{k=1}^{N}{}' \iint_{S_k} \sum_{s=1}^{3} \beta_{sk}^*(p, \xi)\chi_{jskn}(p, \xi, x_n)\, d_\xi S = \frac{(-1)^j \omega_2^2}{4G} N_{jn}^*(p, x_n)$$
$$(j = 1, 2), \quad n = 1, 2, \ldots, N, \quad x_n \in S_n$$
$$\chi_{jskn}(p, \xi, x_n) = \mathbf{L}_{jskn}^{(1)}[\Phi_1(x_{kn}, \xi)] + \mathbf{L}_{jskn}^{(2)}[\Phi_2(x_{kn}, \xi)] \quad (j = 1, 2, 3)$$

The prime on the summation sign here denotes that the term with number $k = n$ is omitted, x_{kn} is the point x_n with coordinates $x_{1kn}, x_{2kn}, x_{3kn}$ in the k-th local coordinate system $O_k x_{1k} x_{2k} x_{3k}$ and $\mathbf{L}_{jskn}$ are differential operators whose coefficients depend on the elastic constants of the material and the geometrical parameters characterizing the location of the slits, determined by means of the formulas (δ_{js} is the Kronecker delta)

$$\mathbf{L}_{jskn}^{(1)} = \left[\delta_{1s}\frac{\partial^2}{\partial x_{2kn}\partial x_{3kn}} - \delta_{2s}\frac{\partial^2}{\partial x_{1kn}\partial x_{3kn}} - \delta_{3s}\left(\Delta_{kn} - \frac{\omega_2^2}{2}\right)\right] \times$$
$$\left[\frac{\nu\omega_1^2}{1 - 2\nu}\delta_{3j} + l_{j3kn}\frac{\partial^2}{\partial x_{1kn}^2} + m_{j3kn}\frac{\partial^2}{\partial x_{2kn}^2} + n_{j3kn}\frac{\partial^2}{\partial x_{3kn}^2} + \right.$$
$$\left. l_{j3kn}^{(1)}\frac{\partial^2}{\partial x_{1kn}\partial x_{2kn}} + m_{j3kn}^{(1)}\frac{\partial^2}{\partial x_{2kn}\partial x_{3kn}} + n_{j3kn}^{(1)}\frac{\partial^2}{\partial x_{1kn}\partial x_{3kn}}\right]$$
$$\mathbf{L}_{jskn}^{(2)} = \delta_{3s}\left\{(\Delta_{kn} - \omega_2^2)\left[l_{j3kn}\frac{\partial^2}{\partial x_{1kn}^2} + m_{j3kn}\frac{\partial^2}{\partial x_{2kn}^2} + n_{j3kn}\frac{\partial^2}{\partial x_{3kn}^2} + \right.\right.$$
$$\left. l_{j3kn}^{(1)}\frac{\partial^2}{\partial x_{1kn}\partial x_{2kn}} + m_{j3kn}^{(1)}\frac{\partial^2}{\partial x_{2kn}\partial x_{3kn}} + n_{j3kn}^{(1)}\frac{\partial^2}{\partial x_{1kn}\partial x_{3kn}}\right] +$$
$$\left.\omega_2^2\left[\frac{1}{2} l_{j3kn}^{(1)}\frac{\partial^2}{\partial x_{1kn}\partial x_{2kn}} + \frac{1}{2} m_{j3kn}^{(1)}\frac{\partial^2}{\partial x_{2kn}\partial x_{3kn}} + n_{j3kn}\frac{\partial^2}{\partial x_{3kn}^2}\right]\right\} +$$
$$(-1)^{s+1}(1 - \delta_{3s})\left\{\left[\left(\Delta_{kn} - \frac{\omega_2^2}{2}\right)^2 - \left(\Delta_{kn} - \frac{3\omega_2^2}{4}\right)\frac{\partial^2}{\partial x_{skn}^2}\right] \times\right.$$
$$(\delta_{1s} m_{j3kn}^{(1)} + \delta_{2s} n_{j3kn}^{(1)}) + \left(\Delta_{kn} - \frac{3\omega_2^2}{4}\right)\frac{\partial^2}{\partial x_{1kn}\partial x_{2kn}} \times$$
$$(\delta_{2s} m_{j3kn}^{(1)} + \delta_{1s} n_{j3kn}^{(1)}) + l_{j3kn}^{(1)}\left(\frac{\omega_2^2}{2} - \frac{\partial^2}{\partial x_{(3-s)kn}^2}\right)\frac{\partial^2}{\partial x_{3kn}\partial x_{skn}} +$$
$$\left[l_{j3kn}\left(\delta_{2s}\frac{\omega_2^2}{2} - \frac{\partial^2}{\partial x_{1kn}^2}\right) + m_{j3kn}\left(\delta_{1s}\frac{\omega_2^2}{2} - \frac{\partial^2}{\partial x_{2kn}^2}\right) + \right.$$
$$\left. n_{j3kn}\left(\frac{\omega_2^2}{2} - \frac{\partial^2}{\partial x_{3kn}^2}\right)\right]\frac{\partial^2}{\partial x_{3kn}\partial x_{(3-s)kn}}, \quad \Delta_{kn} = \frac{\partial^2}{\partial x_{1kn}^2} + \frac{\partial^2}{\partial x_{2kn}^2}$$

Applying an inverse Laplace transform to (1.2), we obtain the integral equations in the originals to determine the functions $\beta_{jn}(t, x)$. They are obtained from (1.2) if convolution theory is used; however, they are of a more awkward structure than (1.2).

The integral equations of the static problem of elasticity theory for an infinite body with plane arbitrarily located cracks /4/ follow as a special case from (1.2) if the $\beta_{jn}^*(p, x)$ in (1.2) are considered to be independent of the parameter p and ω_1 and ω_2 tend to zero. To obtain the integral equations of the problem from (1.6) in the case of steady vibrations, i.e. when the external load has the form $N_{jn}(t, x) = N_{jn}^*(x)\exp(-ikt)$, the ω_j in (1.2) must be replaced by $-ik_j$ where $k_j = k/c_j$ and k is the oscillation frequency.

2. Consider the problem of the steady vibrations of an infinite body due to N plane slits placed in one plane. We consider the surfaces of the slits to be subjected to just the self-equilibrated normal external forces $(N_{3n}(t, x) = N_{3n}^*(x)\exp(-ikt),\ N_{1n}(t, x) = N_{2n}(t, x) = 0)$. The problem of determining the functions β_{3n} that characterize the slit openings reduces to a system of N integral equations (the functions $\beta_{1n}(t, x)$ and $\beta_{2n}(t, x)$ are zero in the case under consideration). Starting from (1.2), this system of equations can be represented in the form (k is the oscillation frequency)

$$\sum_{m=1}^{N}\iint_{S_m}\beta_{3m}(\xi)\Big[K_{1mn}(x_n,\xi)\frac{\exp(ik_1|x_{mn}-\xi|)}{|x_{mn}-\xi|^5} - K_{2mn}(x_n,\xi)\frac{\exp(ik_2|x_{mn}-\xi|)}{|x_{mn}-\xi|^5}\Big]d_\xi S = \frac{k_2^2}{4G}N_{3n}^*(x_n) \tag{2.1}$$

$$x_n\in S_n,\quad n = 1, 2, \ldots, N,\quad K_{1mn}(x_n,\xi) = 9 - 9ik_1|x_{mn}-\xi| - (5k_1^2-k_2^2)|x_{mn}-\xi|^2 + ik_1(2k_1^2-k_2^2)|x_{mn}-\xi|^3 + {}^1/_4(2k_1^2-k_2^2)^2|x_{mn}-\xi|^4$$

$$K_{2mn}(x_n,\xi) = 9 - 9ik_2|x_{mn}-\xi| - 4k_2^2|x_{mn}-\xi|^2 + ik_2^3|x_{mn}-\xi|^3$$

$$k_j = k/c_j$$

If the origins of the local coordinate systems are placed, respectively, in the domains $S_n\ (n = 1, 2, \ldots, N)$ while the axes O_nx_{jn} and O_mx_{jm} are parallel then

$$x_{jmn} = d_{mn}e_{jmn} + x_{jn}\quad (j = 1, 2)$$

where d_{mn} is the distance between the origins of the m-th and n-th coordinate systems, e_{jmn} are the direction cosines of the vector d_{mn} in the m-th coordinate system.

To solve (2.1), we convert them to the form

$$\iint_{S_n}\beta_{3n}(\xi)\Big[\frac{1}{|x_n-\xi|^3} + \frac{Ak_2^2}{|x_n-\xi|} + K(x_n,\xi)\Big]d_\xi S + \frac{4(1-\nu)}{k_2^2}\sum_{m=1}^{N}{}'\iint_{S_m}\beta_{3m}(\xi)\Big[K_{1mn}(x_n,\xi)\frac{\exp(ik_1|x_{mn}-\xi|)}{|x_{mn}-\xi|^5} - K_{2mn}(x_n,\xi)\frac{\exp(ik_2|x_{mn}-\xi|)}{|x_{mn}-\xi|^5}\Big]d_\xi S = \frac{1-\nu}{G}N_{3n}^*(x_n) \tag{2.2}$$

$$x_n\in S_n,\quad n = 1, 2, \ldots, N$$

$$A = \frac{[12(1-\nu)^2 - 8(1-\nu)(1-2\nu) + 3(1-2\nu)^2]}{8(1-\nu)}$$

$$K(x_n,\xi) = \frac{2}{k_2^2-k_1^2}\Big[K_{1nn}(x_n,\xi)\frac{\exp(ik_1|x_n-\xi|)}{|x_n-\xi|^5} - K_{2nn}(x_n,\xi)\frac{\exp(ik_2|x_n-\xi|)}{|x_n-\xi|^5}\Big] - \frac{1}{|x_n-\xi|^3} - \frac{Ak_2^2}{|x_n-\xi|}$$

It can be seen that $K(\xi, \xi) = 0$. To evaluate $K(x_n, \xi)$ for values of x_n close to ξ, we can use the expansion

$$K(x_n,\xi) = \sum_{m=0}^{\infty}A_m|x_n-\xi|^m, \tag{2.3}$$

$$A_m = 4(1-\nu)\frac{i^{m+1}k_2^{m+3}}{(m+1)!}\Big[\frac{1}{4}\Big(\frac{c_2}{c_1}\Big)^{1+m} + \frac{m+2}{m+3}\Big(\frac{c_2}{c_1}\Big)^{3+m} + \frac{m^2+6m+8}{(m+3)(m+5)}\Big(\frac{c_2}{c_1}\Big)^{5+m} + \frac{m+2}{(m+3)(m+5)}\Big]$$

Construction of the approximate analytic solution of system (2.2) by using expansion (2.3) is possible only for low oscillation frequencies and is fraught with serious mathematical difficulties.

3. As an illustration we examine the case of circular disc-shaped slits. We will seek the solution of (2.2) in the form

$$\beta_{3n}(x_n) = \sqrt{a_n^2 - x_{1n}^2 - x_{2n}^2}\,\alpha_n(x_n) \tag{3.1}$$

where a_n is the radius of the n-th disc-shaped slit, and $\alpha_n(x_n)$ are unknown twice-differentiable functions to be determined. Using the method from /5/, we convert the integral equations (2.2) for determining the functions $\alpha_n(x_n)$ to the form

$$\begin{aligned}
&\alpha_n(x_n)\left[I_{00n}(x_n) + Ak_2^2 J_n(x_n)\right] + I_{10n}(x_n)\frac{\partial\alpha_n(x_n)}{\partial x_{1n}} + I_{01n}(x_n)\frac{\partial\alpha_n(x_n)}{\partial x_{2n}} + \\
&\frac{1}{2} I_{20n}(x_n)\frac{\partial^2\alpha_n(x_n)}{\partial x_{1n}^2} + \frac{1}{2} I_{02n}(x_n)\frac{\partial^2\alpha_n(x_n)}{\partial x_{2n}^2} + I_{11n}(x_n)\frac{\partial^2\alpha_n(x_n)}{\partial x_{1n}\partial x_{2n}} + \\
&\iint\limits_{S_n}\frac{\sqrt{a_n^2-\xi_1^2-\xi_2^2}}{|x_n-\xi|^3}\Big\{\alpha_n(\xi) - \alpha_n(x_n) - (\xi_1 - x_{1n})\frac{\partial\alpha_n(x_n)}{\partial x_{1n}} - \\
&(\xi_2 - x_{2n})\frac{\partial\alpha_n(x_n)}{\partial x_{2n}} - \frac{(\xi_1-x_{1n})^2}{2}\frac{\partial^2\alpha_n(x_n)}{\partial x_{1n}^2} - \frac{(\xi_2-x_{2n})^2}{2}\frac{\partial^2\alpha_n(x_n)}{\partial x_{2n}^2} - \\
&(\xi_1-x_{1n})(\xi_2-x_{2n})\frac{\partial^2\alpha_n(x_n)}{\partial x_{1n}\partial x_{2n}} + Ak_2^2|x_n-\xi|^2[\alpha_n(\xi)-\alpha_n(x_n)] + \\
&|x_n-\xi|^3 K(x_n,\xi)\,\alpha_n(\xi)\Big\}\,d_\xi S + \frac{4(1-\nu)}{k_2^2}\sum_{m=1}^{N}{}' \iint\limits_{S_m}\sqrt{a_m^2-\xi_1^2-\xi_2^2}\,\alpha_m(\xi)\times \\
&\Big[K_{1mn}(x_n,\xi)\frac{\exp(ik_1|x_{mn}-\xi|)}{|x_{mn}-\xi|^5} - \\
&K_{2mn}(x_n,\xi)\frac{\exp(ik_2|x_{mn}-\xi|)}{|x_{mn}-\xi|^5}\Big]\,d_\xi S = \frac{1-\nu}{G}N_{3n}^*(x_n), \quad x_n \in S_n \\
&n = 1, 2, \ldots, N \\
&J_n(x_n) = \iint\limits_{S_n}\frac{\sqrt{a_n^2-\xi_1^2-\xi_2^2}}{|x_n-\xi|}\,d_\xi S; \\
&I_{jmn}(x_n) = \iint\limits_{S_n}\frac{\sqrt{a_n^2-\xi_1^2-\xi_2^2}\,(\xi_1-x_{1n})^j(\xi_2-x_{2n})^m}{|x_n-\xi|^3}\,d_\xi S
\end{aligned} \tag{3.2}$$

The integrals J_n, J_{jmn} are evaluated by using the change of variables

$$\xi_1 = x_{1n} + \rho\cos\varphi, \quad \xi_2 = x_{2n} + \rho\sin\varphi.$$

The fact that the integrands vanish for $x_n = \xi$ is used in the numerical solution of the integral Eqs.(3.2).

Having determined the functions $\alpha_n(x_n)$ by means of the appropriate formulas, we find the stress distribution in a body with slits. The stress intensity factors in the neighbourhood of the slits are determined directly in terms of the functions $\alpha_n(x_n)$. For the problem under consideration $K_{2n}(\varphi_n, t) = K_{3n}(\varphi_n, t) = 0$ (φ_n is the angular coordinate of a point on the slit contour in the n-th coordinate system)

$$K_{1n}(\varphi_n, t) = \frac{2G\pi\sqrt{\pi a_n}}{1-\nu}\,\alpha_n(a_n, \varphi_n)\exp(-ikt) \tag{3.3}$$

The dependence $\varkappa = |K_{1n}|/K_1$ (K_1 is the static stress intensity coefficient for one slit under the effect of a load $N_{3n}^*(x_n)$) on its surface) on the angular coordinate φ_n is shown in Figs. 2–4 for different values of the vibrations frequency when there is a system of two and three disc-shaped cracks of unit radius located in one plane. The diagram of the crack location and the measurement of the angle φ_n are indicated in the upper part of the figures. It is assumed that all the slits are loaded by identical normal forces $N_{3n}(t, x_n) = \exp(-ikt)$. The functions $\alpha_n(a_n, \varphi_n)$ were determined by numerical solution of the system (2.2). Curves 1–4 correspond to the values $k/c_2 = 0.2, 0.6, 1.0, 1.2$.

The solid lines in Fig.2 correspond to a distance between the centres of two disc-shaped slits of 2.5a (a is the slit radius). The dashed lines are for the case when this distance is 3a.

The dependences are shown in Fig.3 by solid lines for the left slit and by dash-dot lines for the central slit.

The curves in Fig.4 correspond to the case of three cracks whose centres are at the vertices of an equilateral triangle with side 2.5a.

It follows from the graphs and formulas (3.3) that the stress intensity coefficients K_{1n} at separate times exceed the analogous values determined within the framework of a static formulation of the problem under the same external forces. It follows from the graphs that the most probable rupture direction of the body with slits under consideration subjected to such external loads is on the line connecting the centres of the cracks.

As the oscillation frequency increases, the quantity $|K_{1n}|$ first increases, and then decreases on reaching certain values of k. This value depends on the number of cracks and their location in the plane.

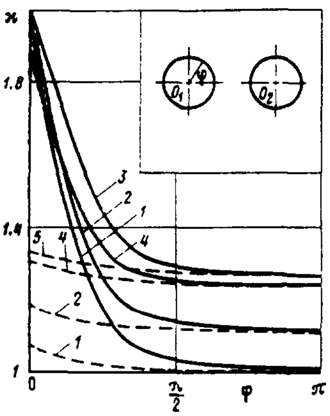

Fig.2

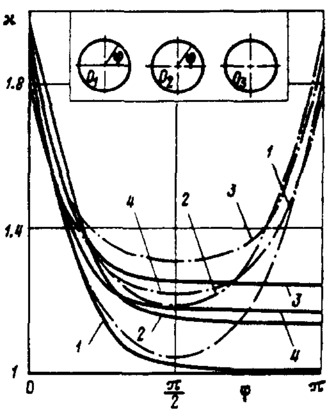

Fig.3

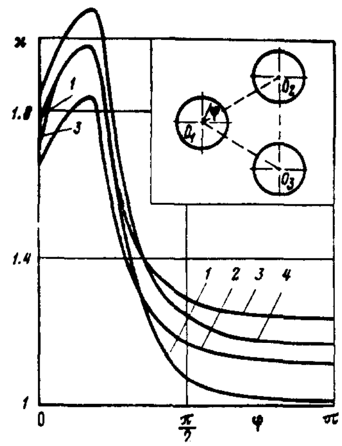

Fig.4

REFERENCES

1. KHAI M.V., On the reduction of three-dimensional dynamical problems of elasticity theory for a body with a crack to integral equations, Mathematical Methods and Physicomechanical Fields, Vyp. 12, Naukova Dumka, Kiev, 1980.
2. GOL'DSHTEIN R.V. and KAPTSOV A.V., Integrodifferential equations of the non-stationary dynamical spatial problem of a crack in an elastic medium, Izv. Akad. Nauk SSSR, Mekhan. Tverd. Tela, 5, 1983.
3. KUPRADZE V.D., GEGELIA T.G., BASHELEISHVILI M.O. and BURCHULADZE T.V., Three-dimensional Problems of the Mathematical Theory of Elasticity and Thermoelasticity. Nauka, Moscow, 1976.
4. KIT G.S. and KHAI M.V., Integral equations of three-dimensional problems of theermo-elasticity for bodies with cracks, Dokl. Akad. Nauk UkrSSR, Ser. A., 12, 1975.
5. KHAI M.V. and KALYNIAK I.V., On an approach to the numerical solution of problems of the mathematical theory of cracks. Mathematical Methods and Physicomechanical Fields, 20, Naukova Dumka, Kiev, 1984.

Translated by M.D.F.